

# Shell-model study of exotic Sn isotopes with a realistic effective interaction

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**Abstract.** We report on a shell-model study of Sn isotopes beyond  $N = 82$  employing a realistic effective interaction derived from the CD-Bonn nucleon-nucleon potential renormalized through use of the  $V_{\text{low-k}}$  approach. At present, the most exotic Sn isotope for which some experimental information exists is  $^{134}\text{Sn}$  with an  $N/Z$  ratio of 1.68. It is the aim of our study to compare the results of our calculations with the available experimental data and to make predictions for the neighboring heavier isotopes which may be within reach of the next generation of radioactive ion beam facilities. The very good agreement between theory and experiment obtained for  $^{134}\text{Sn}$  gives confidence in the predictive power of our realistic shell-model calculations.

## 1. Introduction

The study of Sn isotopes beyond  $N = 82$  is currently the subject of great interest, as it allows to explore for possible changes of the shell-model structure when moving towards the neutron drip line. At present, however, some experimental information is only available for  $^{134}\text{Sn}$ . Beyond this nucleus, which has an  $N/Z$  ratio of 1.68, lies a neutron-rich *terra incognita* which may become accessible to the next generation of radioactive ion beam facilities. In this perspective, it is challenging to make predictions which may stimulate, and provide guidance to, future experiments. In fact, over the past several years various shell-model studies have been performed [1, 2, 3, 4] predicting spectroscopic properties of hitherto unknown neutron-rich Sn isotopes. In these studies both empirical and realistic effective interactions have been used, the latter being based on the CD-Bonn nucleon-nucleon ( $NN$ ) potential model. In this connection, it should be mentioned that a main feature of the study of Ref. [2] is that the short-range repulsion of the bare potential is renormalized by use of the  $V_{\text{low-k}}$  approach [5].

A peculiar property of  $^{134}\text{Sn}$  is the position of its first  $2^+$  state which, lying at 726 keV excitation energy, is the lowest first-excited  $2^+$  level observed in a semi-magic even-even nucleus over the whole chart of nuclides. This feature as well as the other observed excited states -  $4^+$ ,  $6^+$ , and  $8^+$  - have been successfully reproduced by the realistic calculations of [2]. In that study a very good agreement was also obtained with the experimental  $B(E2; 0^+ \rightarrow 2_1^+)$  value measured via Coulomb excitation of a radioactive  $^{134}\text{Sn}$  beam [6].

Very recently, two sophisticated experimental studies [7, 8] have provided new information of great relevance to the study of Sn isotopes beyond  $N = 82$ . More precisely, in [7] a high-

precision Penning trap mass measurement has revealed a 0.5 MeV discrepancy with respect to previous  $Q_\beta$  measurements. This finding provides clear evidence of the robustness of the  $N = 82$  shell closure. In [8], the magic nature of  $^{132}\text{Sn}$  has been explored through the study of the single-particle states in  $^{133}\text{Sn}$  populated by a (d,p) reaction in inverse kinematics. This study has evidenced the purity of the single-neutron excitations in  $^{133}\text{Sn}$  and has identified a strong candidate for the  $2p_{1/2}$  single-particle state at 1.363 MeV excitation energy. This is about 300 keV lower than the previous accepted value [9].

Based on the above, we have found it interesting to revisit our previous realistic shell-model calculations for  $^{134}\text{Sn}$  and extend them to the heavier isotopes up to  $A = 140$ . In this paper we first give a brief description of the theoretical framework in which our calculations are performed and then present and discuss our results which are, for the most part, predictions for possible future experiments. A short summary is given in the last section.

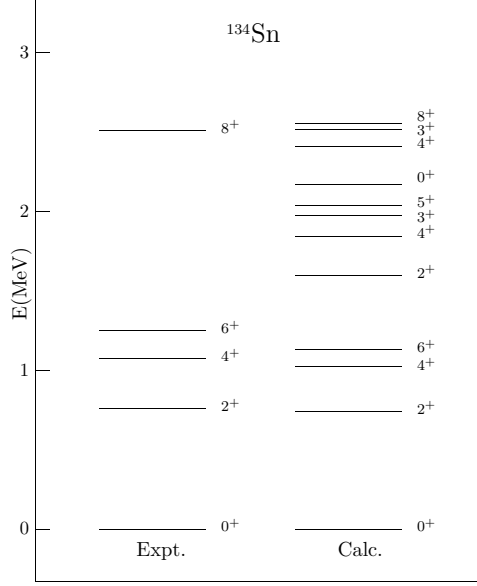
## 2. Outline of calculations

We have performed shell-model calculations for even tin isotopes beyond the  $N = 82$  shell closure, namely, from  $^{134}\text{Sn}$  to  $^{140}\text{Sn}$ . We have taken doubly magic  $^{132}\text{Sn}$  as closed core and let the valence neutrons occupy the six levels  $0h_{9/2}$ ,  $1f_{7/2}$ ,  $1f_{5/2}$ ,  $2p_{3/2}$ ,  $2p_{1/2}$ , and  $0i_{13/2}$  of the  $82 - 126$  shell. Our adopted single-particle (SP) energies relative to the  $1f_{7/2}$  level are (in MeV):  $\epsilon_{p_{3/2}} = 0.854$ ,  $\epsilon_{p_{1/2}} = 1.363$ ,  $\epsilon_{h_{9/2}} = 1.561$ ,  $\epsilon_{f_{5/2}} = 2.005$ , and  $\epsilon_{i_{13/2}} = 2.694$ . These values are taken from the experimental spectrum of  $^{133}\text{Sn}$  [8, 10], with the exception of the  $0i_{13/2}$  level which was determined from the position of the 2.434 MeV level in  $^{134}\text{Sb}$  assumed to be a  $10^+$  state of  $\pi g_{7/2}\nu i_{13/2}$  nature. As for the  $2p_{1/2}$  level, we use the energy reported in the recent experiment of Ref. [8]. It is worth emphasizing that in this experiment the spectroscopic factors of the four levels  $1f_{7/2}$ ,  $2p_{3/2}$ ,  $2p_{1/2}$ ,  $1f_{5/2}$  were extracted, which provide a clear confirmation of their SP nature. The absolute energy of the  $1f_{7/2}$  level relative to  $^{132}\text{Sn}$  was placed at  $-2.371$  MeV, as resulting from the new mass measurement of  $^{133}\text{Sn}$  [7].

The shell-model effective interaction has been derived within the framework of perturbation theory [11] starting, as mentioned in the Introduction, from the CD-Bonn  $NN$  potential [12] renormalized by way of the so-called  $V_{\text{low-k}}$  approach [5]. More precisely, we start by deriving  $V_{\text{low-k}}$  with a cutoff momentum of  $\Lambda = 2.2 \text{ fm}^{-1}$ . Then, using this potential plus the Coulomb force for protons, we calculate the two-body matrix elements of the effective interaction by means of the  $\hat{Q}$ -box folded-diagram expansion, with the  $\hat{Q}$ -box including all diagrams up to second order in the interaction. These diagrams are computed within the harmonic-oscillator basis using intermediate states composed of all possible hole states and particle states restricted to the five proton and neutron shells above the Fermi surface. The oscillator parameter is 7.88 MeV for the  $A = 132$  region, as obtained from the expression  $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ . The calculations have been performed by using the OSLO shell-model code [13].

## 3. Results

We start this section by discussing our results for  $^{134}\text{Sn}$ , the only tin isotope beyond  $N = 82$  for which some experimental information is available. In figure 1 all the observed levels [10] are reported and compared with the calculated ones up to about 2.5 MeV excitation energy. The experimental spectrum consists of a group of low-lying levels separated by an energy gap of about 1.5 MeV from the highest observed level which has  $J^\pi = 8^+$ . The former correspond to the four calculated members of the  $\nu(f_{7/2})^2$  multiplet while the latter to the maximum-aligned state of the  $\nu f_{7/2}h_{9/2}$  configuration and all energies are very well reproduced by the theory. In the energy interval between 1.1 and 2.5 MeV our calculation predicts the existence of other states, namely the members of the  $\nu f_{7/2}p_{3/2}$  and  $\nu f_{7/2}p_{1/2}$  multiplets, and of the  $0^+$  state of the  $\nu(p_{3/2})^2$  configuration. It is worth noting that in our previous calculation [2] the  $3_2^+$  and  $4_3^+$



**Figure 1.** Experimental and calculated spectrum of  $^{134}\text{Sn}$ .

states were predicted to lie above the  $8^+$  state, their downshift in energy being directly related to the new position of the  $p_{1/2}$  SP level.

We have also calculated some electromagnetic properties and our predicted values for the first four excited states are reported in table 1, where the two experimental transition rates presently available [6, 10] are also shown. Our calculations have been performed using an effective neutron charge of  $0.7e$  [1], which leads to  $B(E2)$  values for both the  $6^+ \rightarrow 4^+$  and  $2^+ \rightarrow 0^+$  transitions quite close to the experimental ones.

**Table 1.** Electromagnetic transition rates (in W.u.), quadrupole (in  $\text{efm}^2$ ) and magnetic moments (in nm) in  $^{134}\text{Sn}$ .

	Calc	Expt
$B(E2; 2_1^+ \rightarrow 0_1^+)$	1.64	$1.42 \pm 0.20$
$B(E2; 4_1^+ \rightarrow 2_1^+)$	1.66	
$B(E2; 6_1^+ \rightarrow 4_1^+)$	0.82	$0.89 \pm 0.17$
$B(E2; 2_2^+ \rightarrow 0_1^+)$	0.35	
$B(E2; 2_2^+ \rightarrow 2_1^+)$	2.93	
$B(E2; 2_2^+ \rightarrow 4_1^+)$	0.23	
$B(M1; 2_2^+ \rightarrow 2_1^+)$	0.02	
$Q(2_1^+)$	-1.6	
$Q(2_2^+)$	-2.8	
$\mu(2_1^+)$	-0.57	
$\mu(2_2^+)$	-0.25	

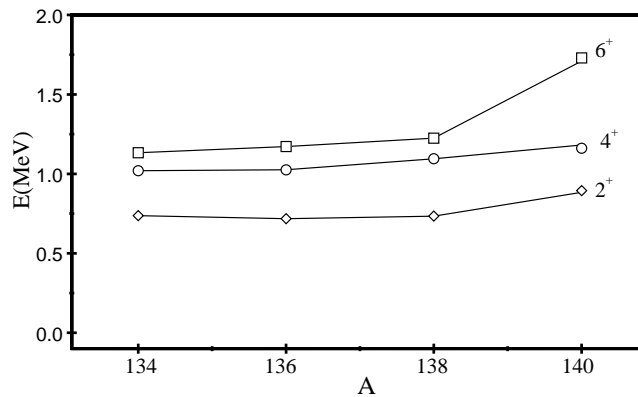
We hope that the above predictions for  $^{134}\text{Sn}$  may provide guidance to future experiments aiming at the identification of the missing states above the yrast  $6^+$  state as well as at the

measurement of other properties, which may certainly give better insight into the structure of this exotic nucleus.

In this context, it should be mentioned that several interesting questions have been posed by the new, although still scarce, experimental data which have become available for some nuclei in the  $^{132}\text{Sn}$  region. In particular, special attention has been focused on the properties of the yrast  $2^+$  states of tin and tellurium isotopes, whose excitation energy shows an asymmetric behavior with respect to  $N = 82$ . In fact, in  $^{134}\text{Sn}$  and  $^{136}\text{Te}$  the  $2_1^+$  energy undergoes a significant drop as compared to the  $N = 80$  isotopes, which was traced to a reduction of the neutron pairing above the  $N = 82$  shell [14, 15].

With our realistic effective interaction we are able, as shown above, to reproduce with good accuracy the energy of the yrast  $2^+$  state and its  $B(E2)$  transition rate in  $^{134}\text{Sn}$ . This was also the case for  $^{136}\text{Te}$  (see [16]) whose  $2^+$  state is described as the corresponding state in  $^{134}\text{Sn}$  with the two additional protons coupled to zero angular momentum. As a matter of fact, the  $2^+$  state, as well as the other three lowest-lying states in  $^{134}\text{Sn}$ , are characterized by a very weak configuration mixing. Therefore the small  $0^+ - 2^+$  spacing reflects the small difference between the  $\langle (f_{7/2})^2 | V_{\text{eff}} | (f_{7/2})^2 \rangle_{0^+}$  and  $\langle (f_{7/2})^2 | V_{\text{eff}} | (f_{7/2})^2 \rangle_{2^+}$  matrix elements, which turns out to be about 0.4 MeV. This result is due to the negligible contribution [17] of the one particle-one hole excitations in the effective interaction of two neutrons above the  $N = 82$  shell, which are instead responsible for the increase of pairing below this shell. The weakness of the cross-shell interaction for  $^{132}\text{Sn}$  as compared with other lighter closed cores was also pointed out in Ref. [3], where it was noted that the excitation energy of the  $3^-$  state in  $^{132}\text{Sn}$  is only 0.5 MeV smaller than the shell gap for neutrons.

We have investigated the evolution of the yrast  $2^+$  state and of the other  $J \neq 0$  members of the lowest-lying multiplet when adding neutron pairs. In figure 2 we report the excitation energies of the  $2_1^+$ ,  $4_1^+$ , and  $6_1^+$  states as a function of  $A$  up to  $^{140}\text{Sn}$ . We see that the curves are almost flat, with a slight increase at  $N = 90$  for the  $2_1^+$  and  $4_1^+$  which becomes more significant for the  $6^+$  state.



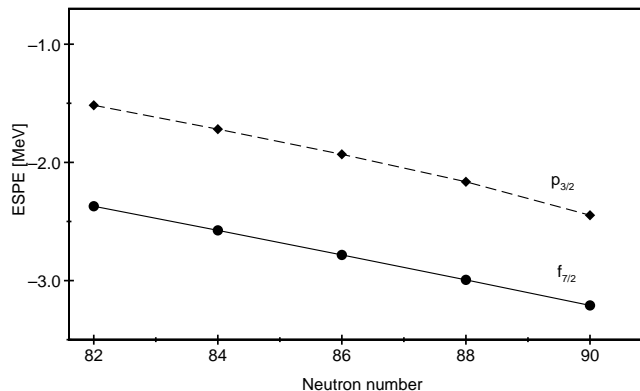
**Figure 2.** Calculated excitation energies of the yrast  $2^+$ ,  $4^+$ , and  $6^+$  states in tin isotopes with  $A = 134, 136, 138$ , and  $140$ .

This result is similar to that found in [3], the main difference being that our  $4_1^+$  and  $6_1^+$  curves lie about 100-300 MeV below those of [3], while is in contrast to the conclusion of the recent work of [4] claiming for a shell closure at  $N = 90$ . The results of [4] predict an upshift of the  $2^+$  state in  $^{140}\text{Sn}$  with respect to the lighter isotopes when using an empirical interaction (SMPN) as well as a realistic interaction with three-body monopole corrections (CWG3M). This upshift was related to the filling of the  $f_{7/2}$  orbit at  $N = 90$  as a consequence of a significant increase

**Table 2.** Binding energies and one-neutron separation energies (in MeV) in tin isotopes with  $A = 134, 136, 138,$  and  $140$ .

	$^{134}\text{Sn}$	$^{136}\text{Sn}$	$^{138}\text{Sn}$	$^{140}\text{Sn}$
BE Calc	5.92	11.83	17.68	23.41
BE Expt	$5.916 \pm 0.150$			
$S_n$ Calc	3.55	3.55	3.53	3.50
$S_n$ Expt	$3.545 \pm 0.152$			

in the difference between the effective single-particle energies of the  $f_{7/2}$  and  $p_{3/2}$  orbits which, starting from 0.854 keV, becomes larger than 2 MeV at  $A = 140$ . As can be seen in figure 3, our calculations do not predict such an increase. The energy difference between the two orbits remains in fact almost constant and even decreases slightly by about 100 keV when moving from  $N = 82$  to  $N = 90$ . As regards the  $6^+$  state, we find that its wave function in  $^{140}\text{Sn}$  is dominated by the  $\nu(f_{7/2})^6(p_{3/2})^2$  configuration, which explains its increasing in energy.



**Figure 3.** Effective single-particle energies of the  $f_{7/2}$  and  $p_{3/2}$  levels from  $N = 82$  to  $N = 90$ .

In concluding this section, we discuss binding (relative to  $^{132}\text{Sn}$ ) and one-neutron separation energies of the even Sn isotopes. The calculated values are reported in table 2, where the experimental available values for  $^{134}\text{Sn}$  are also shown. As mentioned in the Introduction, a new measurement of the binding energy of  $^{134}\text{Sn}$  has revealed a 0.5 MeV deviation from the previous observed value. With the new mass value the maximum of the neutron-shell gap for  $N=82$  is at  $Z = 50$ , instead of  $Z = 51$  as established by the old measurement. We find an excellent agreement between theory and experiment, which shows the reliability of the  $J = 0$  matrix elements of our effective interaction. This confirms the weakening of pairing for two neutrons beyond the  $N = 82$  shell. As regards the heavier Sn isotopes, we see that the binding energy keeps increasing with increasing mass number, while the one-neutron separation energy remains practically constant. Our calculations, in line with those of [3] and the estimations of mean field calculations, seem to indicate that in this region we are still quite far from the neutron drip line.

#### 4. Summary

We have presented here the results of a shell-model study of exotic even mass Sn isotopes from  $A = 134$  to  $A = 140$ . In our calculations we have employed a realistic low-momentum effective

interaction derived from the CD-Bonn  $NN$  potential without using any adjustable parameter.

The heaviest Sn isotope for which there is some, albeit scarce, experimental information is  $^{134}\text{Sn}$ , so most of our results are predictive in nature. We have shown that the available experimental data are very well reproduced by our calculations. This gives confidence in their predictive power, which we hope can be verified by experiment in the not too distant future.

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